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This manuscript provides a step-by-step guide to statistical power calculation for the fixed effects of analysis of variance (ANOVA) designs with an equal number of observations in each cell. A brief history of ANOVA hypothesis testing theory is included to explain why power calculation is important and how the results can be used. The relationship between lambda (λ), the noncentrality parameter used to calculate power in the ANOVA, and Cohen's (1988) measure of effect size is provided. Algorithms are provided for power calculation and for conversion between λ, Cohen's measure of effect size, and <u>phi</u>--the parameter used in many tables of the noncentral <u>F</u> distribution. The appendices contain power of calculation examples for the main and interaction effects of 2 x 3 x 3 between- and within- subjects designs.

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A Computational Guide to Power Analysis of Fixed Effects in Balanced Analysis of Variance Designs

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September 1995

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Director's Foreword

The results of numerous published studies, both within and outside of the psychophysiological detection of deception (PDD) literature, are based on observation groups which are too small to provide results that are representative of the general population. Such studies are described as having low or insufficient statistical power. These publications not only represent a misuse of potentially useful resources, but may lead to unjustified, if not erroneous, conclusions. Among the potential reasons for the prevalence of such studies in the literature are the limited awareness of statistical power analysis, and difficulty associated with the calculation of statistical power.

This manuscript is the first of several computational guides to statistical power analysis to be developed at the Institute. It is designed to assist the investigator in designing, and understanding the analysis of, fixed effects in balanced factorial analysis of variance statistical designs. Future guides will address statistical power calculation for the commonly used student-$\underline{t}$ and chi-square inferential statistics. This and future documents should assist others, as they have the DoDPI faculty, in both the design and evaluation of PDD investigations.

John R. Schwartz
Acting Director

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Abstract

DOLLINS, A. B. A computational guide to power analysis of fixed effects in balanced analysis of variance designs, September 1995, Report No. DoDPI95-R-0003. Department of Defense Polygraph Institute, Ft. McClellan, AL 36205.--This manuscript provides a step-by-step guide to statistical power calculation for the fixed effects of analysis of variance (ANOVA) designs with an equal number of observations in each cell. A brief history of ANOVA hypothesis testing theory is included to explain why power calculation is important and how the results can be used. The relationship between lambda (λ), the noncentrality parameter used to calculate power in the ANOVA, and Cohen's (1988) measure of effect size is provided. Algorithms are provided for power calculation and for conversion between λ, Cohen's measure of effect size, and *phi*--the parameter used in many tables of the noncentral *F* distribution. The appendices contain power calculation examples for the main and interaction effects of 2 x 3 x 3 between- and within-subjects designs.

Key Words: Computation guide, analysis of variance (ANOVA), statistical power, lambda (λ), alpha (α), beta (β), effect size, algorithm.

Executive Summary

DOLLINS, A. B. A computational guide to power analysis of fixed effects in balanced analysis of variance designs, September 1995, Report No. DoDPI95-R-0003. Department of Defense Polygraph Institute, Ft. McClellan, AL 36205.

The power of a statistical test is the probability that the test will correctly reject the null hypothesis. Statistical power is commonly used to calculate the number of observations necessary to yield statistically significant results or to calculate the probability that a statistically significant effect would have been found if one existed. The power of a statistical test should not be confused with the significance of a statistical test--which is the probability that a true null hypothesis is falsely rejected. It is possible to obtain statistically significant effects with low or high power. Most text books concerning statistics describe power calculation procedures, but they are usually brief, sometimes difficult to understand, and occasionally misleading. This manuscript is an attempt to provide a clear, easy to understand, step-by-step guide to the calculation of statistical power for fixed effects of analysis of variance (ANOVA) designs with an equal number of observations in each cell. A brief history of ANOVA hypothesis testing theory is presented to explain why power calculation is important and how its results can be used. The confusing issue of whether a hypothesis may only be rejected, versus rejected or accepted, is explained. The relationship between lambda (λ), the noncentrality parameter used to calculate power in the ANOVA, and Cohen's (1988) measure of effect size is provided. Algorithms are provided for power calculation and for conversion between λ, Cohen's measure of effect size, and *phi*--the parameter used in many tables of the noncentral *F* distribution. The appendices contain power calculation examples for the main and interaction effects of 2 x 3 x 3 between- and within-subjects designs.

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Scheffé (1959, p. 3) roughly defines the analysis of variance (ANOVA) as "a statistical technique for analyzing measurements depending on several kinds of effects operating simultaneously, to decide which kinds of effects are important and to estimate the effects." Scheffé (1959, p. 3) attributes the development of ANOVA techniques chiefly to R. A. Fisher (1918, 1935), who was attempting to address agricultural rather than psychological research.

In practice, the ANOVA is a set of procedures for calculating the probability that a particular set of observations could have occurred by chance (i.e., randomly). Thus, a hypothesis may be rejected, with some degree of confidence, that a similar set of observations would not occur by chance. The hypothesis tested is usually the null hypothesis that two or more means (dependent variables), observed during two or more experimental manipulations (independent variables) are equal. This hypothesis may only be rejected (i.e., the groups of values are not equal) based on the ANOVA of the observed values. It is important to note that failure to reject a hypothesis does not, according to Fisherian logic, indicate acceptance of the hypothesis (Fisher, 1966, p. 16). (Cohen [1990] argues that this is a flaw in Fisherian logic because the null hypothesis is always false in the real world - given a large enough sample size.) If the probability that the observed values could have occurred by chance is less than a preset probability level (i.e., referred to as the significance criterion or <u>alpha</u> [α]), the null hypothesis is rejected.

Neyman and Pearson (1928a, 1928b) proposed that the specification of an alternative hypothesis be added to the ANOVA. (This concept was, according to Cohen [1990, p. 1307], violently opposed by Fisher.) Inclusion of an alternative hypothesis, to be accepted if the null hypothesis was rejected, revolutionized the decision process associated with the ANOVA. Now the ANOVA could be used to both support and reject hypotheses. Including an alternative hypothesis, with an associated effect size, allows the calculation of the probability that the null hypothesis is not rejected when it is false, as well as the probability of rejecting the null hypothesis, given that the alternative hypothesis is true--referred to as beta (β) or Type II error. Calculation of β allows calculation of its compliment (i.e., $1 - \beta$), power, the probability that the null hypothesis is correctly rejected. The probability that the null hypothesis will be rejected when it is true, alpha (α), is referred to as the Type I error rate.

The number of observations necessary to support a hypothesis can thus be calculated--given the desired α and β probabilities and the magnitude of the difference between the null and alternate hypotheses. The power of an ANOVA test can also be calculated--given the desired α level, the number of observations, and the magnitude of the difference between the null and alternate hypotheses. Power analysis is primarily used to determine the probability that a statistically significant difference will be obtained, given a specified difference among the observations, and a specified number of observations; or the probability that a statistically significant effect would have been obtained (where none is found) if one had existed. While it is possible to calculate and use the parameters necessary to support a hypothesis with a relatively high degree of confidence it is, apparently, rarely done. This is documented by the relatively low power (< .60) of the majority of studies, in numerous research fields, to detect small and medium effects (Bones, 1972; Brewer 1972; Brewer & Owne, 1972; Brown & Hale, 1992; Chase & Tucker, 1975; Chase & Chase, 1976; Chase & Barnum, 1976; Christensen & Christensen, 1977; Cohen, 1962; Crane, 1976; Daly & Hexamer, 1983; Fagley, 1985; Frieman, Chalmers, Smith, & Kuebler, 1978; Haase, 1974; Haase, Waechter, Solomon, 1982; Hall, 1982; Jones & Brewer, 1972; Julnes & Mohr, 1989; Kosciulek, 1993; Kosciulek & Szymanski, 1993; Kroll & Chase, 1975; Orme & Combs-Orme, 1986; Orme & Tolman, 1986; Ottenbacher, 1982; Penick & Brewer, 1972; Rossi, 1990; Rothpearl, Mohs, & Davis, 1981; Sawyer & Ball, 1981; Sedlmeier & Gigerenzer, 1989; Wolley, 1983; Wooley & Dawson, 1983). According to S. E. Edgell (personal communication, August 14, 1995) the main problem with low power is that the researcher wastes time by running studies that have little chance of finding the result desired.

Perhaps one of the reasons that the power of $\underline{F}$ ratios are not calculated or reported more frequently is the difficulty associated with power calculations. Calculating the power of $\underline{F}$ ratios in ANOVA designs can be difficult, particularly for designs with more than one factor and/or repeated factors, because the majority of the calculations must be completed by hand. The most complete text on the topic of power analysis is Jacob Cohen's _Statistical Power Analysis for the Behavioral Sciences_ (1988)--which addresses power calculation for most commonly used parametric and non-parametric statistics. Unfortunately, Cohen's (1988) calculations for the ANOVA (pp. 273-406, 550-551) are appropriate for one-way ANOVA designs but underestimate the power and overestimate the sample sizes of higher level designs (Koele, 1982). (Note: Koele was referring to the calculations described in the 1977 edition of Cohen's book - which remain the same in the more current 1988 version.) As can be seen in Appendices B and C, this is true for between-subjects designs, but the reverse is true for within-subjects designs. Cohen (1988) does not describe power calculations for repeated measure ANOVA designs in any detail, but suggestions may be found elsewhere (Bavry, 1991, pp. 63-76; Davidson, 1972, p. 448; Koele, 1982; Kraemer & Thiemann, 1987, pp. 45-52; Lipsey, 1990, pp. 79-84; Winer, 1971, p. 516).

Cohen (1988) does, however, note several important observations concerning power analysis. Statistical significance levels have generally been set by convention to .05 or .01 (Cowles & Davis, 1982). No such convention exists for power levels, however, Cohen (1988, p. 56) suggests that the value of .80 be used when the investigator has no other basis for setting the desired power value. Cohen (1988, pp. 284-288, 355) further proposes that ANOVA effect sizes, for the behavioral sciences, be categorized into small (.10), medium (.25), and large (.40) for theoretical purposes. Cohen (1988, pp. 364-367) also notes that it is possible to calculate power for separate effects of a complex factorial design independently. This is somewhat analogous to the independent calculation of the effects in a complex factorial design.

The following guide to calculating the power of fixed effects in balanced ANOVA design $\underline{F}$ tests is designed to summarize what can be a very confusing process. The works of Bavry (1991), Borenstein and Cohen (1988), Cohen (1988), Koele (1982), and Winer (1971) were relied upon most heavily during the development of this guide. It should be noted that the processes described herein are based primarily on statistical theory rather than empirical evidence. Monte Carlo studies of the statistical power of ANOVA designs have, however, been reported (Cole, Maxwell, Arvey, & Salas, 1994; Cornell, Young, Seaman, & Kirk, 1992; Keselman, Rogan, Mendoze, & Breen, 1980; Klockars & Hancock, 1992). The description below pertains only to power analysis of a complex fixed effect between- and within-subjects factorial ANOVA designs with an equal number of observations in each cell (Cohen's, 1988, case 2). Adjustments for an unequal number of observations in each cell are described by Cohen.

Power Calculation

To calculate the power of the $\underline{F}$ ratio of a complex fixed effect ANOVA design, it is necessary to know the: significance criterion of the $\underline{F}$ ratio for which power is calculated (α); degrees of freedom of the numerator of the $\underline{F}$ ratio for which power is calculated; degrees of freedom of the denominator of the $\underline{F}$ ratio for which power is calculated; and, the noncentrality parameter associated with the $\underline{F}$ ratio. As detailed below, the noncentrality parameter can be calculated using Cohen's effect size--$\underline{f}$ (1988). If predicting the power of a repeated measure design using data from a between-subjects design, it is also necessary to calculate the (assumed constant) correlation between pairs of observations on the same element and factor level, as detailed below.

According to Koele (1982), the power of a fixed effect ANOVA $\underline{F}$ test is the probability that ($\underline{F} > \underline{Fc}$ given $\underline{df1}$, $\underline{df2}$, lambda [λ]). Koele defines λ as the noncentrality parameter;

df1 and df2 as the numerator and denominator, respectively, degrees of freedom of the F ratio for which power is being calculated, and Fc as the critical F value (with df1 and df2 degrees of freedom) that the F ratio must exceed at a given significance level. It is distributed as a noncentral F distribution.

Significance Criterion / Fc - Critical F Value

Fc is the F ratio, associated with a given probability (α) level which the calculated F statistic must exceed to be significantly different from chance. For instance, an observed F ratio with df1 = 3 and df2 = 20 must exceed Fc = 3.10 to be statistically significant at an α level of 0.05 and must exceed Fc = 4.94 to be statistically significant at an α level of 0.01. This value can be calculated from the central F distribution given the α level and the numerator and denominator degrees of freedom. It can also be obtained from tables of the central F distribution given in most textbooks of statistical analyses (e.g. Winer, 1971; Keppel, 1991).

Numerator Degrees of Freedom - df1

These are the degrees of freedom associated with the numerator of the F ratio for which power is calculated.

Denominator Degrees of Freedom - df2

These are the degrees of freedom associated with the denominator of the F ratio for which power is calculated.

The Noncentrality Parameter - λ

The noncentrality parameter is equal to the F statistic numerator sum of squares, with each term replaced by its expectation, divided by the within-cells error variance (i.e., the mean squares error term; Kendall & Stuart, 1966, p. 5; Scheffe, 1959, p. 39). The noncentrality parameter, λ is thus equal to the calculated F ratio times its numerator degrees of freedom. For example, the λ associated with $F(2, 8) = 63.389$ would be 2 * 63.389 or 126.778, and the λ associated with $F(4, 16) = 0.357$ would be 4 * 0.357 or 1.427 (see Appendices A, B, and C for more examples).

The Noncentral F Distribution

Once Fc, df1, df2, and λ are determined, power calculation is completed by use of the noncentral F distribution. Tables of this distribution are provided by Rotton and Schonemann (1978), Tiku (1967), and most textbooks on ANOVA. Table powers are usually indexed by df1, df2, and phi (ϕ)--rather than λ. According to Winer, Brown, and Michels (1991, p. 408), λ can be converted to ϕ using the following algorithm.

$$\phi = \mathrm{SQRT}[\lambda \ / \ (\text{number of effect levels})]$$

Laubscher (1960) describes a square root normal approximation of the noncentral F distribution (formula 6) which may be used to calculate the power of an F ratio using a hand calculator and tables of the central F and Z distributions. While both Cohen (1988, p. 550) and Laubscher (1960) describe a cube root normal approximation, Laubscher concluded that the square root approximation was slightly more accurate for the tested data set. Cohen (1988, p. 550) comments that Laubscher's square root normal approximation of noncentral F "gave excellent agreement with exact value determinations given in the literature...except when n and f are small," but does not define small. A somewhat simplified version of Cohen's adaptation of Laubscher's square root approximation of the probabilities given by the noncentral F distribution is:

$$\underline{X1} = (\underline{df1} + 2 * \lambda) / (\underline{df1} + \lambda)$$
$$\underline{X2} = (\underline{df1} * \underline{Fc}) / \underline{df2}$$

$$\underline{Z} = \frac{\mathrm{SQRT}[2 * (\underline{df1} + \lambda) - \underline{X1}] - \mathrm{SQRT}[(2 * \underline{df2} - 1) * \underline{X2}]}{\mathrm{SQRT}(\underline{X1} + \underline{X2})}$$

$$\text{Power} >= 1 - [\text{Probability of } (\underline{Z})]$$

Where:

$\underline{df1}$ = numerator degrees of freedom of the original F ratio.
$\underline{df2}$ = denominator degrees of freedom of the original F ratio.
λ = the non-centrality parameter.
$\underline{Fc}$ = the value of the critical F ratio given the original F ratio degrees of freedom and significance criterion.
$\underline{Z}$ = A Z value, the probability of which may be determined using a table of proportions of area under the standard normal curve. This probability is the probability of a Type II error (i.e., β).

The following computer programs and associated manuals were used in the preparation of this manuscript: _Statistical design analysis system_ (Bavry, in press); _Stat-Power statistical design analysis system_ (Bavry, 1991); and _Statistical power analysis: A computer program_ (Borenstein & Cohen, 1988). A review of computer programs used to calculate power analyses may be found elsewhere (Goldstein, 1989).

Effect Size

Calculating Effect Size

Calculating the power of a completed F test is thus a relatively straightforward task given the significance criterion, F ratio degrees of freedom, and λ. As mentioned above, however, power analysis is primarily useful in predicting the number of observations needed to obtain a significant effect, if one exists, with a given power, or the probability that a statistically significant effect would have been obtained if one had existed. In both cases, the $\underline{F}$ ratio necessary to predict λ does not exist and must be estimated. The discerning reader will realize that it may be difficult to estimate λ on an a priori basis. Several investigators have proposed ANOVA-based measures of effect size to assist in λ estimation, as reviewed by Tatsuoka (1993). Probably the most intuitive is Cohen's $\underline{f}$, which is defined as the standard deviation of the effect means divided by the (common) within-cell standard deviation (Cohen, 1988, pp. 274-275). While Cohen (1988, pp. 215-406) provides several examples of the standard deviation of the effect means calculations, a detailed explanation of the (common) within-cell standard deviation is not found. Hedges (1981), however, demonstrated that the square root of the $\underline{F}$ ratio's within-cell mean square error term provides the best unbiased estimator of the within-cell standard deviation. Thus, the terminology of Cohen (1988) and Hedges (1981) are adapted as:

$$\text{Effect size } (\underline{f}) = \underline{SDm} \,/\, \underline{SDe}$$

Where:

$\underline{f}$ = Cohen's ANOVA-based effect size (Cohen uses the letter $\underline{f}$ to indicate effect size - this should not be confused with the uppercase $\underline{F}$ which is used to denote the $\underline{F}$ ratio).

$\underline{SDm}$ = The standard deviation of the effect means.

$\underline{SDe}$ = The square root of the within-cell mean square error term.

The effect size numerator ($\underline{SDm}$) is calculated using one of three techniques depending on the type of factor (main effect vs. interaction) and the number of levels. Calculation procedures for the effect size denominator ($\underline{SDe}$) for a between-subjects ANOVA design differs from those for a within-subjects ANOVA design. These are detailed below and numerical examples are provided in Appendix D. Before proceeding with the examples, a short description of the notation used is necessary. The capital letter "M" is used to indicate the mean of a cell, lower case letters are used to indicate the factor, and arabic numbers are used to indicate the factor level. A period will be used to indicate that a particular factor has been averaged. Thus: "Ma.." indicates the means associated with factor A; "Ma1.." indicates the mean of factor A, level 1; "M.b." indicates the means associated with factor B; "M..c" indicates the means associated with factor C; "Mabc" indicates the cell means associated with the A x B x C interaction; and "M..." indicates the grand mean of all values in the data set. For within-subject designs, the notation for specific observations follows the same pattern where: "Ma1..s1" indicates the average of subject 1's scores over level 1 of factor A and "M..c4s3" indicates the average of subject 3's scores over level 4 of factor C.

The following examples are for an A (2 levels) x B (3 levels) x C (4 levels) design with 5 observations per cell. The SDm is calculated in the same manner for both the within- and between-subjects designs. The same SDe term is used to calculate the effect size of each factor in a between-subject design - in the same manner as a common mean square error term is used when calculating the F ratio for each test of a between subjects design. The SDe term is used to calculate the effect size of each factor in a within-subjects design varies, as does the mean square error term used when calculating the F ratios of a within-subjects design.

The SDe term for the A (2 levels) x B (3 levels) x C (4 levels) example with 5 independent observations in each cell is:

$$\text{SDe} = \text{SQRT}\left[\frac{\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{4}\sum_{l=1}^{5} (\text{Xaibjcksl} - \text{Maibjck.})\text{\^{}}2}{2*3*4*(5-1)}\right]$$

Note: ^ = exponentiation, thus X^2 = X*X.

Or, more simply, the square root of the average cell variance:

Factor	SDe
A	SQRT[(VARa1b1c1 + VARa2b1c1 +...+ VARa2b3c4) / 24]
B	SQRT[(VARa1b1c1 + VARa2b1c1 +...+ VARa2b3c4) / 24]
.	SQRT[(VARa1b1c1 + VARa2b1c1 +...+ VARa2b3c4) / 24]
A x B x C	SQRT[(VARa1b1c1 + VARa2b1c1 +...+ VARa2b3c4) / 24]

Where: VAR is the variance.

The general SDe term for a within-subjects ANOVA is the square root of the within-cell mean square error term used in the F ratio for which the power is being calculated. A general example is given below and examples of specific calculations for the various effects may be found in Appendix D:

$$\text{SDe} = \text{SQRT}\left[\frac{\sum_{x=1}^{2} [\ (\sum_{y=1}^{5} (\text{Max..sy} - \text{Max...})\text{\^{}}2\ ]}{8\ (\text{i.e., the } F \text{ ratio denominator } df)}\right]$$

(1) Effect size of a main effect with 2 levels is calculated using:

$$\underline{f} = \left| \frac{0.5 * (\text{maximum Ma..} - \text{minimum Ma..})}{\underline{\text{SDe}}} \right|$$

Note: The standard deviation of two values is 0.5 * the difference between the two values.

(2) Effect size of a main effect with more than 2 levels is calculated using:

$$\underline{f} = \frac{\text{SQRT}\{ [\sum_{x=1}^{\underline{N}} (\text{M..cx} - \text{M...})\text{^}2] / \underline{N} \}}{\underline{\text{SDe}}}$$

(3) Interaction effect sizes are the square root of the summed squares of the contribution of each cell to the effect divided by the number of cells. The contribution of each cell's effect is calculated by removing the contributions of other factors to that cells effect (i.e., using the linear model). The process is similar to that used to calculate the sum of squares for an $\underline{F}$ ratio interaction. For example, the effect size for Cohen's (1988) example 8.6 (pp. 368-372) A(2 levels) x B(3 levels) interaction would be calculated as: .

Xa1b1. = Ma1b1. - Ma1.. - M.b1. + M...
Xa1b2. = Ma1b2. - Ma1.. - M.b2. + M...
Xa1b3. = Ma1b3. - Ma1.. - M.b3. + M...
Xa2b1. = Ma2b1. - Ma2.. - M.b1. + M...
Xa2b2. = Ma2b2. - Ma2.. - M.b2. + M...
Xa2b3. = Ma2b3. - Ma2.. - M.b3. + M...

$$\underline{f} = \frac{\text{SQRT}\{ \sum_{x=1}^{2} [\sum_{y=1}^{3} (\text{Xaxby} \text{ ^ } 2)] / (2*3)\}}{\underline{\text{SDe}}}$$

Note: Calculating the cell contributions can become quite complex. A good guide for the factors and signs may be found in Kirk (1968). The X???s used to calculate the $\underline{\text{SDm}}$ for Cohen's example 8.6 A x B x C effect would be:

Xabc = Mabc - Mab. - Ma.c - M.bc + Ma.. + M.b. + M..c - M...

Converting Cohen's Effect Size to λ

Cohen's (1988) ANOVA-based measure of effect size can be converted to λ using the following algorithm.

λ = *f*^2 * (the total number of observations analyzed for the effect)

The total number of observations analyzed for an effect is the number of observations used in calculating the error term and will differ for within- and between-subjects ANOVA designs. For example, the number of observations for the effects of an A (2 levels) x B (3 levels) x C (4 levels) ANOVA with 5 observations per cell, analyzed as a within- or between-subjects design would be:

Effect	Total Number of Observations Within-subjects	Total Number of Observations Between-subjects
A	10	120
B	15	120
AxB	15	120
C	20	120
AxC	20	120
BxC	120	120
AxBxC	120	120

A Note Concerning Cohen's Description of ANOVA Power Calculation

The power tables for ANOVA designs provided by Cohen (1988, pp. 273-406) require specification of: a desired significance criterion; an effect size; the *F* ratio numerator degrees of freedom; and the sample size. Cohen (1988, p. 365) indicates that it is necessary to use an adjusted samples size to cope with the discrepancy in denominator (error) degrees of freedom between one-way and higher-way ANOVA designs. Cohen (1988, p. 365) describes the calculation of sample size (*n*') as follows:

$$\text{sample size} = n' = \frac{\text{denominator } df}{u + 1} + 1$$

Where:

u = the degrees of freedom associated with the numerator of the *F* ratio for which power is to be calculated.

denominator *df* = total number of observations in the analysis minus the total number of cells in the analysis.

An example calculation of n for each of the effects in a 2(A) x 3(B) x 4(C) ANOVA with 5 observations per cell (Cohen's example 8.6, p. 368-372) would be:

Total observations = 120 (i.e., 2 * 3 * 4 * 5)
Total number of cells = 24 (i.e., 2 * 3 * 4)
Denominator *df* = 120 - 24 = 96

Effect	Numerator *df*	*n*
A	1	49.0
B	2	33.0
C	3	25.0
A x B	2	33.0
A x C	3	25.0
B x C	6	14.7
A x B x C	6	14.7

This adjustment works well for a one-way ANOVA design. However, as noted by Koele (1982), and illustrated in Appendices B and C, using Cohen's technique to calculate the power of effects in higher-way ANOVA designs will result in an underestimation of the power of between-subjects design effects and overestimation of the power of within-subjects effects. It is thus suggested that Cohen's ANOVA-based effect size measure be converted to and/or from λ and noncentral *F* distribution probabilities be used to estimate power. This will ensure accurate results and is, in addition, less complicated computationally.

Constant Correlation

An assumption in repeated measures ANOVA is that there is a "constant" correlation between pairs of observations on the same subject under different conditions (Winer, 1971, p. 516). Winer (1971, p. 516) and others (Lipsey, 1990, pp. 79-84; Davidson, 1972, p. 448; Kraemer & Thiemann, 1987, pp. 45-52) suggest that *SDe* should be increased or decreased according to the constant correlation when attempting to estimate the *SDe* for a within-subjects ANOVA design using existing data from a study with a between-subjects ANOVA design (details below). A problem occurs when deciding how to estimate the constant correlation. When comparing only two observations, the product-moment correlation may be used as an estimate of the constant correlation. Dr. Bavry (personal communication) and others (Silver & Dunlap, 1987; Silver & Hollingsworth, 1989; Viana, 1980, 1993) suggest that the best estimate of the constant correlation is calculated by averaging the Fisher's *Z* transform (Fisher, 1921) of all of the within-subjects between-cell correlations, then converting that Fisher's *Z* transform average back to a correlation coefficient. An numerical example of constant correlation calculation for data presented in Appendix A is given in Appendix E. Fisher's *Z* transform and its inverse are as follows (Silver & Dunlap, 1987).

Fisher's $\underline{Z}$ transform is: $\underline{Z} = 0.5 * \log_e [(1+\underline{r})/(1-\underline{r})]$

The inverse transform is: $\underline{r} = (\underline{X} - 1) / (\underline{X} + 1)$

Where:

$\underline{r}$ = the correlation coefficient
$\underline{X} = \exp_e (2 * \underline{Z})$

Note: The constant correlation correction is only necessary when attempting to estimate the SDe for a within-subjects ANOVA design using existing data from a study with a between-subjects ANOVA design.

According to Winer (1971, p. 516), the following correction should be used to adjust estimates of SDe obtained from between-subjects designs when calculating power analyses of $\underline{F}$ ratios involving repeated measures. The SDe of repeated measure interaction and main effects should be adjusted by multiplying SDe by (1-r), where r is the constant correlation for that effect. The SDe of between groups effects which are composed of repeated measures on each member of a group should be adjusted by multiplying SDe by $(1 + \underline{W} * \underline{r})$, where $\underline{W}$ is the tested effect degrees of freedom and r is the constant correlation for that effect.

Description of the Appendices

Appendix A contains the results of between-subjects and within-subjects ANOVA of data presented by Winer (1962, p. 324; 1971, p. 546). Appendices B and C contain the results of a power analysis of the data in Appendix A using the suggested noncentral $\underline{F}$ distribution and Cohen's tables, respectively. A comparison of the results obtained using the two methods illustrates the tendency of Cohen's technique to overestimate between-subjects and underestimate within-subjects higher-way ANOVA effect powers. Appendix D contains a numerical example of the calculations necessary to obtain the data presented in Appendices B and C. Appendix E contains a numerical example of the use of Fisher's $\underline{Z}$ transform to calculate the average correlation of data in Appendix A. Appendix F contains algorithms for converting values among ϕ, λ, and Cohen's effect size for ANOVA ($\underline{f}$).

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Appendix A

Example Data and Analyses of Variance

The Raw Data (Winer, 1962, p. 324, 1971, p. 546):

Subject	Noise	Period	P1			P2			P3		
		Dial	D1	D2	D3	D1	D2	D3	D1	D2	D3
1	N1		45	53	60	40	52	57	28	37	46
2	N1		35	41	50	30	37	47	25	32	41
3	N1		60	65	75	58	54	70	40	47	50
4	N2		50	48	61	25	34	51	16	23	35
5	N2		42	45	55	30	37	43	22	27	37
6	N2		56	60	77	40	39	57	31	29	46

Results of the analysis of Winer's data (1962, p. 324; 1971, p. 546) as a 2(NOISE-between) x 3(PERIOD-within) x 3(DIAL-within) ANOVA

BETWEEN SUBJECTS

SOURCE	SS	DF	MS	F	P
NOISE	468.167	1	468.167	0.752	0.435
ERROR	2491.111	4	622.778		

WITHIN SUBJECTS

SOURCE	SS	DF	MS	F	P	G-G	H-F
PERIOD	3722.333	2	1861.167	63.389	0.000	0.000	0.00
NOISE*PERIOD	333.000	2	166.500	5.671	0.029	0.057	0.02
ERROR	234.889	8	29.361				
GREENHOUSE-GEISSER EPSILON:			0.6476	HUYNH-FELDT EPSILON:			1.0000
DIAL	2370.333	2	1185.167	89.823	0.000	0.000	0.00
NOISE*DIAL	50.333	2	25.167	1.907	0.210	0.215	0.21
ERROR	105.556	8	13.194				
GREENHOUSE-GEISSER EPSILON:			0.9171	HUYNH-FELDT EPSILON:			1.0000
PERIOD*DIAL	10.667	4	2.667	0.336	0.850	0.729	0.85
NOISE*PERIOD*DIAL	11.333	4	2.833	0.357	0.836	0.716	0.83
ERROR	127.111	16	7.944				
GREENHOUSE-GEISSER EPSILON:			0.5134	HUYNH-FELDT EPSILON:			1.0000

Results of the analysis of Winer's data (1962, p. 324; 1971, p. 546) as a 2(NOISE-between) x 3(PERIOD-between) x 3(DIAL-between) ANOVA

SOURCE	SS	DF	MS	F	P
NOISE	468.167	1	468.167	5.696	0.022
PERIOD	3722.333	2	1861.167	22.646	0.000
DIAL	2370.333	2	1185.167	14.421	0.000
NOISE*PERIOD	333.000	2	166.500	2.026	0.147
NOISE*DIAL	50.333	2	25.167	0.306	0.738
PERIOD*DIAL	10.667	4	2.667	0.032	0.998
NOISE*PERIOD*DIAL	11.333	4	2.833	0.034	0.998
ERROR	2958.667	36	82.185		

Appendix B

Power Calculations using the noncentral F distribution

POWER of Winer's (1962, p. 324; 1971, p. 546) 2(NOISE-between) x 3(PERIOD-within) x 3(DIAL-within) ANOVA example using Bavry's Non-central Cumulative *F* Probability calculation.

Factor	df Numerator	df Denominator	λ	Non-central Cumulative Probability (β)	Power ($1 - \beta$)
NOISE	1	4	0.752	0.896	0.104 +
PERIOD	2	8	126.778	0.000	0.999
NOISE*PERIOD	2	8	11.342	0.303	0.697
DIAL	2	8	179.652	0.000	0.999
NOISE*DIAL	2	8	3.815	0.714	0.286 +
PERIOD*DIAL	4	16	1.343	0.893	0.107 +
NOISE*PERIOD*DIAL	4	16	1.427	0.889	0.111 +

POWER of Winer's (1962, p. 324; 1971, p. 546) 2(NOISE-between) x 3(PERIOD-between) x 3(DIAL-between) ANOVA data using Bavry's Non-central Cumulative *F* Probability calculation.

Factor	df Numerator	df Denominator	λ	Non-central Cumulative Probability (β)	Power ($1 - \beta$)
NOISE	1	36	5.697	0.358	0.642
PERIOD	2	36	45.292	0.000	1.000
DIAL	2	36	28.841	0.002	0.998
NOISE*PERIOD	2	36	4.052	0.610	0.390 +
NOISE*DIAL	2	36	0.612	0.905	0.095 +
PERIOD*DIAL	4	36	0.130	0.944	0.056 +
NOISE*PERIOD*DIAL	4	36	0.138	0.944	0.056 +

+ These power values are given to illustrate the use of the cited formulae. They are not indicative of the power of the original *F* ratio because the original *F* ratio did not reach significance at the 0.05 level.

Appendix C

Power Calculations using Cohen's tables and technique

POWER of Winer's (1962, p. 324; 1971, p. 546) 2(NOISE-between) x 3(PERIOD-within) x 3(DIAL-within) ANOVA example using Cohen's method.

	u	n/group	n'	SDm	SDe	f	POWER
NOISE	1.000	3.000	3.000	2.944	8.318	0.354	0.111 +
PERIOD	2.000	6.000	6.000	8.302	3.129	2.653	0.999
NOISE*PERIOD	2.000	6.000	6.000	2.483	3.129	0.794	0.705 +
DIAL	2.000	6.000	6.000	6.625	2.097	3.159	0.999
NOISE*DIAL	2.000	6.000	6.000	0.965	2.097	0.460	0.337 +
PERIOD*DIAL	4.000	9.800	10.800	0.444	2.818	0.157	0.117 +
NOISE*PERIOD*DIAL	4.000	9.800	10.800	0.458	2.818	0.163	0.123 +

POWER of Winer's (1962, p. 324; 1971, p. 546) 2(NOISE-between) x 3(PERIOD-between) x 3(DIAL-between) ANOVA data using Cohen's method.

	u	n/group	n'	SDm	SDe	f	POWER
NOISE	1.000	27.000	19.000	2.944	9.065	0.324	0.573
PERIOD	2.000	18.000	13.000	8.302	9.065	0.916	0.999
DIAL	2.000	18.000	13.000	6.625	9.065	0.731	0.980
NOISE*PERIOD	2.000	9.000	13.000	2.483	9.065	0.274	0.293 +
NOISE*DIAL	2.000	9.000	13.000	0.965	9.065	0.106	0.078 +
PERIOD*DIAL	4.000	6.000	8.200	0.444	9.065	0.049	0.051 +
NOISE*PERIOD*DIAL	4.000	3.000	8.200	0.458	9.065	0.051	0.053 +

+ These power values are given to illustrate the use of the cited formulae. They are not indicative of the power of the original F ratio because the original F ratio did not reach significance at the 0.05 level.

Appendix D

Numerical Examples of Power Calculation

Calculation of SDe for the data from Winer's (1962, p. 324; 1971, p. 546) 2 x 3 x 3 example analyzed as a 2(NOISE-between) x 3(PERIOD-between) x 3(DIAL-between) design.

N#P#D#S				
1 1 1 .	Mn=	46.6667	Var=	158.3333
1 1 2 .	Mn=	53.0000	Var=	144.0000
1 1 3 .	Mn=	61.6667	Var=	158.3333
1 2 1 .	Mn=	42.6667	Var=	201.3333
1 2 2 .	Mn=	47.6667	Var=	86.3333
1 2 3 .	Mn=	58.0000	Var=	133.0000
1 3 1 .	Mn=	31.0000	Var=	63.0000
1 3 2 .	Mn=	38.6667	Var=	58.3333
1 3 3 .	Mn=	45.6667	Var=	20.3333
2 1 1 .	Mn=	49.3333	Var=	49.3333
2 1 2 .	Mn=	51.0000	Var=	63.0000
2 1 3 .	Mn=	64.3333	Var=	129.3333
2 2 1 .	Mn=	31.6667	Var=	58.3333
2 2 2 .	Mn=	36.6667	Var=	6.3333
2 2 3 .	Mn=	50.3333	Var=	49.3333
2 3 1 .	Mn=	23.0000	Var=	57.0000
2 3 2 .	Mn=	26.3333	Var=	9.3333
2 3 3 .	Mn=	39.3333	Var=	34.3333
Grand Average	=	44.2777	Var=	82.1852

SDe = SQRT(82.1852) = 9.0656

Power Calculation for NOISE Effect:

Calculation of SDm for the NOISE effect:

N#P#D#S#	Xi	=Mn...	-M....
1 . . .	2.944	=47.222	44.278
2 . . .	-2.944	=41.333	44.278

Xi	Xi^2	Sum of Squares (Xi^2) * (Observations/average)
2.944	8.670	234.083
-2.944	8.670	234.083
	SUM = 17.340	SUM = 468.167

$$SDm = \mathrm{Sqrt}\left(\sum_{i=1}^{2} (Xi^2) / I\right) = \mathrm{SQRT}(17.340 / 2) = 2.944$$

Calculation of SDe for the NOISE effect:

N#P#D#S#	Yj	=Mn..s	-Mn...
1 . . 1	-0.778	46.444	47.222
1 . . 2	-9.667	37.556	47.222
1 . . 3	10.444	57.667	47.222
2 . . 4	-3.222	38.111	41.333
2 . . 5	-3.778	37.556	41.333
2 . . 6	7.000	48.333	41.333

Yj	Yj^2	Sum of Squares (Yj^2) * (Observations/average)
-0.778	0.605	5.444
-9.667	93.444	841.000
10.444	109.086	981.778
-3.222	10.383	93.444
-3.778	14.272	128.444
7.000	49.000	441.000
	SUM = 276.790	SUM =2491.111

$$SDe = \mathrm{Sqrt}\left(\sum_{j=1}^{6} (Yj^2) / \text{denominator } df\right) = \mathrm{SQRT}(276.790 / 4) = 8.318$$

COHEN	BAVRY
Effect Size = f = SDm / SDe	λ = SSm / MSe
Effect Size = f = 2.944 / 8.318	λ = 468.167 / (2491.111/4)
Effect Size = 0.354	λ = 0.752
u = 1.0	DFm = 1.0
n'= (4 / (1 + 1)) + 1 = 3	DFe = 4.0
Effect power = 0.111	Effect power = 0.104

Note: Cohen's Denominator df = (6/2 - 1) * 2 = 4

Power Calculation for the PERIOD and NOISE*PERIOD Effects:

Calculation of SDm for the PERIOD effect:

N#P#D#S#	Xi	=M.p..	-M....
. 1 . .	10.055	54.333	44.278
. 2 . .	0.222	44.500	44.278
. 3 . .	-10.278	34.000	44.278

Xi	Xi^2	Sum of Squares (Xi^2) * (Observations/average)
10.055	101.103	1819.854
0.222	0.049	.882
-10.278	105.637	1901.466
	SUM = 206.789	SUM = 3722.202

SDm = $\mathrm{Sqrt}\left(\sum_{i=1}^{3} (Xi^2) / I \right) = \mathrm{SQRT}(206.789 / 3) = 8.302$

Calculation of SDm for the NOISE*PERIOD effect:

N#P#D#S#	Xi	=Mnp..	-M.p..	-Mn...	+M....
1 1 . .	-3.4995	53.778	54.333	47.222	44.277
1 2 . .	2.0001	49.444	44.500	47.222	44.277
1 3 . .	1.5001	38.444	34.000	47.222	44.277
2 1 . .	3.5006	54.889	54.333	41.333	44.277
2 2 . .	-1.9997	39.556	44.500	41.333	44.277
2 3 . .	-1.4997	29.556	34.000	41.333	44.277

Xi	Xi^2	Sum of Squares (Xi^2) * (Observations/average)
-3.499	12.246	110.219
2.000	4.000	36.004
1.500	2.250	20.253
3.500	12.254	110.288
-1.999	3.999	35.989
-1.499	2.249	20.242
	SUM = 36.999	SUM = 332.9937

SDm = $\mathrm{Sqrt}\left(\sum_{i=1}^{6} (Xi^2) / I \right) = \mathrm{SQRT}(36.999 / 6) = 2.483$

Calculation of SDe for the PERIOD and NOISE*PERIOD Effects:

N#P#D#S#	Yj	=Mnp.s	-Mnp..	-Mn..s	+Mn...
1 1 . 1	-0.333	52.667	53.778	46.445	47.222
1 1 . 2	-2.111	42.000	53.778	37.556	47.222
1 1 . 3	2.444	66.667	53.778	57.667	47.222
2 1 . 4	1.333	53.000	54.889	38.111	41.333
2 1 . 5	-3.778	47.333	54.889	37.556	41.333
2 1 . 6	2.445	64.333	54.889	48.333	41.333
1 2 . 1	1.000	49.667	49.445	46.445	47.222
1 2 . 2	-1.778	38.000	49.445	37.556	47.222
1 2 . 3	0.778	60.667	49.445	57.667	47.222
2 2 . 4	0.333	36.667	39.556	38.111	41.333
2 2 . 5	0.889	36.667	39.556	37.556	41.333
2 2 . 6	-1.222	45.333	39.556	48.333	41.333
1 3 . 1	-0.667	37.000	38.445	46.445	47.222
1 3 . 2	3.889	32.667	38.445	37.556	47.222
1 3 . 3	-3.222	45.667	38.445	57.667	47.222
2 3 . 4	-1.667	24.667	29.556	38.111	41.333
2 3 . 5	2.889	28.667	29.556	37.556	41.333
2 3 . 6	-1.222	35.333	29.556	48.333	41.333

Yj	Yj^2	Sum of Squares (Yj^2) * (Observations/average)
-0.333	0.111	0.333
-2.111	4.457	13.372
2.444	5.975	17.926
1.333	1.778	5.333
-3.778	14.273	42.820
2.445	5.976	17.929
1.000	1.000	3.001
-1.778	3.161	9.483
0.778	0.605	1.815
0.333	0.111	0.333
0.889	0.790	2.371
-1.222	1.494	4.482
-0.667	0.445	1.334
3.889	15.125	45.375
-3.222	10.383	31.148
-1.667	2.778	8.333
2.889	8.346	25.039
-1.222	1.494	4.482
	SUM = 78.303	SUM = 234.910

$$\underline{SDe} = \mathrm{Sqrt}\left(\sum_{j=1}^{18} (Yj^2) \,/\, \text{denominator } \underline{df} \right) = \mathrm{SQRT}(78.303 / 8) = 3.129$$

Power Calculation for the PERIOD Effect:

COHEN

Effect Size = f = SDm / SDe

Effect Size = f = 8.302 / 3.129

Effect Size = 2.653

u = 2.0

n' = (15 / (2 + 1)) + 1 = 6.00

Effect power = 1.000 (0.9999)

BAVRY

λ = SSm / MSe

λ = 3722.333 / (234.889/8)

λ = 126.777

DFm = 2.0

DFe = 8.0

Effect power = 1.000 (0.9999)

Power Calculation for the NOISE*PERIOD Effect:

COHEN

Effect Size = f = SDm / SDe

Effect Size = f = 2.483 / 3.129

Effect Size = 0.794

u = 2.0

n' = (15 / (2 + 1)) + 1 = 6.00

Effect power = 0.7055

BAVRY

λ = SSm / MSe

λ = 333.000 / (234.889/8)

λ = 11.342

DFm = 2.0

DFe = 8.0

Effect power = 0.6973

Note: Denominator df = (18/3 - 1) * 3 = 15

Power Calculation for the DIAL and NOISE*DIAL Effects:

Calculation of SDm for the DIAL effect:

N#P#D#S#	Xi	=M..d.	-M....
. . 1 .	-6.889	37.389	44.278
. . 2 .	-2.056	42.222	44.278
. . 3 .	8.944	53.222	44.278

Xi	Xi^2	Sum of Squares (Xi^2) * (Observations/average)
-6.889	47.458	854.250
-2.056	4.227	76.088
8.944	79.995	1439.912
	SUM = 131.681	SUM = 2370.251

$$\underline{SDm} = \text{Sqrt}\left(\sum_{i=1}^{3} (Xi^2) / I \right) = \text{SQRT}(131.681 / 3) = 6.625$$

Calculation of SDm for the NOISE*DIAL effect:

N#P#D#S#	Xi	=Mn.d.	-M..d.	-Mn...	+M....
1 . 1 .	-0.221	40.111	37.389	47.222	44.278
1 . 2 .	1.278	46.444	42.222	47.222	44.278
1 . 3 .	-1.055	55.111	53.222	47.222	44.278
2 . 1 .	0.222	34.667	37.389	41.333	44.278
2 . 2 .	-1.277	38.000	42.222	41.333	44.278
2 . 3 .	1.056	51.333	53.222	41.333	44.278

Xi	Xi^2	Sum of Squares (Xi^2) * (Observations/average)
-0.221	0.049	0.439
1.278	1.634	14.702
-1.055	1.113	10.021
0.222	0.050	0.446
-1.277	1.632	14.684
1.056	1.115	10.036
	SUM = 5.592	SUM = 50.326

$$\underline{SDm} = \text{Sqrt}\left(\sum_{i=1}^{6} (Xi^2) / I \right) = \text{SQRT}(5.592 / 6) = 0.965$$

Calculation of SDe for the DIAL and NOISE*DIAL Effects:

N#P#D#S#	Yj	=Mn.ds	-Mn.d.	-Mn..s	+Mn...
1 . 1 1	-1.667	37.667	40.111	46.444	47.222
1 . 1 2	-0.444	30.000	40.111	37.556	47.222
1 . 1 3	2.111	52.667	40.111	57.667	47.222
2 . 1 4	-1.111	30.333	34.667	38.111	41.333
2 . 1 5	0.444	31.333	34.667	37.556	41.333
2 . 1 6	0.667	42.333	34.667	48.333	41.333
1 . 2 1	1.667	47.333	46.444	46.444	47.222
1 . 2 2	-0.111	36.667	46.444	37.556	47.222
1 . 2 3	-1.556	55.333	46.444	57.667	47.222
2 . 2 4	0.222	35.000	38.000	38.111	41.333
2 . 2 5	2.111	36.333	38.000	37.556	41.333
2 . 2 6	-2.333	42.667	38.000	48.333	41.333
1 . 3 1	0.000	54.333	55.111	46.444	47.222
1 . 3 2	0.556	46.000	55.111	37.556	47.222
1 . 3 3	-0.556	65.000	55.111	57.667	47.222
2 . 3 4	0.889	49.000	51.333	38.111	41.333
2 . 3 5	-2.556	45.000	51.333	37.556	41.333
2 . 3 6	1.667	60.000	51.333	48.333	41.333

Yj	Yj^2	Sum of Squares (Yj^2) * (Observations/average)
-1.667	2.778	8.333
-0.444	0.198	0.593
2.111	4.457	13.370
-1.111	1.235	3.704
0.444	0.198	0.593
0.667	0.444	1.333
1.667	2.778	8.333
-0.111	0.012	0.037
-1.556	2.420	7.259
0.222	0.049	0.148
2.111	4.457	13.370
-2.333	5.444	16.333
0.000	0.000	0.000
0.556	0.309	0.926
-0.556	0.309	0.926
0.889	0.790	2.370
-2.556	6.531	19.593
1.667	2.778	8.333
	SUM = 35.185	SUM = 105.556

$$\underline{SDe} = \mathrm{Sqrt}\left(\sum_{j=1}^{18} (Yj\text{^}2) \;/\; \text{denominator } \underline{df}\right) = \mathrm{SQRT}(35.185 / 8) = 2.097$$

Power Calculation for the DIAL Effect:

COHEN

Effect Size = f = SDm / SDe
Effect Size = f = 6.625 / 2.097
Effect Size = 3.159
u = 2.0
n' = (15 / (2 + 1)) + 1 = 6.00
Effect power = 1.000 (0.9999)

BAVRY

λ = SSm / MSe
λ = 2370.333 / (105.556/8)
λ = 179.652
DFm = 2.0
DFe = 8.0
Effect power = 1.000 (0.9999)

Power Calculation for the NOISE*DIAL Effect:

COHEN

Effect Size = f = SDm / SDe
Effect Size = f = 0.965 / 2.097
Effect Size = 0.460
u = 2.0
n' = (15 / (2 + 1)) + 1 = 6.00
Effect power = 0.337

BAVRY

λ = SSm / MSe
λ = 50.333 / (105.556/8)
λ = 3.815
DFm = 2.0
DFe = 8.0
Effect power = 0.286

Note : Denominator df = (18/3 - 1) * 3 = 15

Power Calculation for the PERIOD*DIAL and NOISE*PERIOD*DIAL Effects:

Calculation of SDm for the PERIOD*DIAL effect:

N#P#D#S#	Xi	=M.pd..	-M..d..	-M.p...	+M....
. 1 1 .	0.556	48.000	37.389	54.333	44.278
. 1 2 .	-0.277	52.000	42.222	54.333	44.278
. 1 3 .	-0.277	63.000	53.222	54.333	44.278
. 2 1 .	-0.444	37.167	37.389	44.500	44.278
. 2 2 .	-0.277	42.167	42.222	44.500	44.278
. 2 3 .	0.723	54.167	53.222	44.500	44.278
. 3 1 .	-0.111	27.000	37.389	34.000	44.278
. 3 2 .	0.556	32.500	42.222	34.000	44.278
. 3 3 .	-0.444	42.500	53.222	34.000	44.278

Xi	Xi^2	Sum of Squares (Xi^2) * (Observations/average)
0.556	0.309	1.855
-0.277	0.077	0.460
-0.277	0.077	0.460
-0.444	0.197	1.183
-0.277	0.077	0.460
0.723	0.523	3.136
-0.111	0.012	0.074
0.556	0.309	1.855
-0.444	0.197	1.183
	SUM = 1.778	SUM = 10.667

$$SDm = \mathrm{Sqrt}\left(\sum_{i=1}^{9} (Xi^2) / I \right) = \mathrm{SQRT}(1.778 / 9) = 0.444$$

Calculation of SDm for the NOISE*PERIOD*DIAL effect:

N#P#D#S#	Xi	= Mnpd.	-Mnp..	-Ma.d.	-M.pd.	+Mn...	+M.p..	+M..d.	-M....
1 1 1 .	-0.555	46.667	53.778	40.111	48.000	47.222	54.333	37.389	44.277
1 1 2 .	0.278	53.000	53.778	46.444	52.000	47.222	54.333	42.222	44.277
1 1 3 .	0.278	61.667	53.778	55.111	63.000	47.222	54.333	53.222	44.277
1 2 1 .	0.779	42.667	49.444	40.111	37.167	47.222	44.500	37.389	44.277
1 2 2 .	-0.721	47.667	49.444	46.444	42.167	47.222	44.500	42.222	44.277
1 2 3 .	-0.055	58.000	49.444	55.111	54.167	47.222	44.500	53.222	44.277
1 3 1 .	-0.221	31.000	38.444	40.111	27.000	47.222	34.000	37.389	44.277
1 3 2 .	0.446	38.667	38.444	46.444	32.500	47.222	34.000	42.222	44.277
1 3 3 .	-0.221	45.667	38.444	55.111	42.500	47.222	34.000	53.222	44.277
2 1 1 .	0.555	49.333	54.889	34.667	48.000	41.333	54.333	37.389	44.277
2 1 2 .	-0.278	51.000	54.889	38.000	52.000	41.333	54.333	42.222	44.277
2 1 3 .	-0.278	64.333	54.889	51.333	63.000	41.333	54.333	53.222	44.277
2 2 1 .	-0.778	31.667	39.556	34.667	37.167	41.333	44.500	37.389	44.277
2 2 2 .	0.722	36.667	39.556	38.000	42.167	41.333	44.500	42.222	44.277
2 2 3 .	0.055	50.333	39.556	51.333	54.167	41.333	44.500	53.222	44.277
2 3 1 .	0.222	23.000	29.556	34.667	27.000	41.333	34.000	37.389	44.277
2 3 2 .	-0.445	26.333	29.556	38.000	32.500	41.333	34.000	42.222	44.277
2 3 3 .	0.222	39.333	29.556	51.333	42.500	41.333	34.000	53.222	44.277

Xj	Xj^2	Sum of Squares (Xj^2) * (Observations/average)
-0.555	0.308	0.924
0.278	0.077	0.232
0.278	0.077	0.232
0.779	0.607	1.821
-0.721	0.520	1.560
-0.055	0.003	0.009
-0.221	0.049	0.147
0.446	0.199	0.597
-0.221	0.049	0.147
0.555	0.308	0.924
-0.278	0.077	0.232
-0.278	0.077	0.232
-0.778	0.605	1.816
0.722	0.521	1.564
0.055	0.003	0.009
0.222	0.049	0.148
-0.445	0.198	0.594
0.222	0.049	0.148
	SUM = 3.778	SUM = 11.333

$$\underline{SDm} = \mathrm{Sqrt}\left(\sum_{i=1}^{18} (Xi^2) / I \right) = \mathrm{SQRT}(3.778 / 18) = 0.458$$

Calculation of SDe for the PERIOD*DIAL and NOISE*PERIOD*DIAL Effects:

N#P#D#S#	Yj^2	Yi =	Mnpds	-Mnpd.	-Mnp.s	-Mn.ds	+Mnp..	+Mn.d.	+Mn..s	-Mn...
1 1 1 1	1.234	1.111	45.000	46.666	52.667	37.667	53.778	40.111	46.444	47.222
1 1 1 2	0.310	0.557	35.000	46.666	42.000	30.000	53.778	40.111	37.556	47.222
1 1 1 3	2.776	-1.666	60.000	46.666	66.667	52.667	53.778	40.111	57.667	47.222
2 1 1 4	13.454	3.668	50.000	49.333	53.000	30.333	54.889	34.667	38.111	41.333
2 1 1 5	0.048	-0.220	42.000	49.333	47.333	31.333	54.889	34.667	37.556	41.333
2 1 1 6	11.854	-3.443	56.000	49.333	64.333	42.333	54.889	34.667	48.333	41.333
1 1 2 1	0.309	-0.556	53.000	53.000	52.667	47.333	53.778	46.444	46.444	47.222
1 1 2 2	0.012	-0.111	41.000	53.000	42.000	36.667	53.778	46.444	37.556	47.222
1 1 2 3	0.445	0.667	65.000	53.000	66.667	55.333	53.778	46.444	57.667	47.222
2 1 2 4	1.777	-1.333	48.000	51.000	53.000	35.000	54.889	38.000	38.111	41.333
2 1 2 5	0.307	-0.554	45.000	51.000	47.333	36.333	54.889	38.000	37.556	41.333
2 1 2 6	3.568	1.889	60.000	51.000	64.333	42.667	54.889	38.000	48.333	41.333
1 1 3 1	0.309	-0.556	60.000	61.667	52.667	54.333	53.778	55.111	46.444	47.222
1 1 3 2	0.197	-0.444	50.000	61.667	42.000	46.000	53.778	55.111	37.556	47.222
1 1 3 3	1.000	1.000	75.000	61.667	66.667	65.000	53.778	55.111	57.667	47.222
2 1 3 4	5.443	-2.333	61.000	64.333	53.000	49.000	54.889	51.333	38.111	41.333
2 1 3 5	0.607	0.779	55.000	64.333	47.333	45.000	54.889	51.333	37.556	41.333
2 1 3 6	2.421	1.556	77.000	64.333	64.333	60.000	54.889	51.333	48.333	41.333
1 2 1 1	1.498	-1.224	40.000	42.667	49.667	37.667	49.444	40.111	46.444	47.222
1 2 1 2	0.605	-0.778	30.000	42.667	38.000	30.000	49.444	40.111	37.556	47.222
1 2 1 3	3.996	1.999	58.000	42.667	60.667	52.667	49.444	40.111	57.667	47.222
2 2 1 4	7.113	-2.667	25.000	31.667	36.667	30.333	39.555	34.667	38.111	41.333
2 2 1 5	0.605	0.778	30.000	31.667	36.667	31.333	39.555	34.667	37.556	41.333
2 2 1 6	3.568	1.889	40.000	31.667	45.333	42.333	39.555	34.667	48.333	41.333
1 2 2 1	5.968	2.443	52.000	47.667	49.667	47.333	49.444	46.444	46.444	47.222
1 2 2 2	0.789	0.888	37.000	47.667	38.000	36.667	49.444	46.444	37.556	47.222
1 2 2 3	11.116	-3.334	54.000	47.667	60.667	55.333	49.444	46.444	57.667	47.222
2 2 2 4	0.000	-0.001	34.000	36.667	36.667	35.000	39.555	38.000	38.111	41.333
2 2 2 5	1.234	1.111	37.000	36.667	36.667	36.333	39.555	38.000	37.556	41.333
2 2 2 6	1.237	-1.112	39.000	36.667	45.333	42.667	39.555	38.000	48.333	41.333
1 2 3 1	1.496	-1.223	57.000	58.000	49.667	54.333	49.444	55.111	46.444	47.222
1 2 3 2	0.012	-0.111	47.000	58.000	38.000	46.000	49.444	55.111	37.556	47.222
1 2 3 3	1.777	1.333	70.000	58.000	60.667	65.000	49.444	55.111	57.667	47.222
2 2 3 4	7.108	2.666	51.000	50.333	36.667	49.000	39.555	51.333	38.111	41.333
2 2 3 5	3.568	-1.889	43.000	50.333	36.667	45.000	39.555	51.333	37.556	41.333
2 2 3 6	0.605	-0.778	57.000	50.333	45.333	60.000	39.555	51.333	48.333	41.333
1 3 1 1	0.012	0.110	28.000	31.000	37.000	37.667	38.444	40.111	46.444	47.222
1 3 1 2	0.049	0.222	25.000	31.000	32.667	30.000	38.444	40.111	37.556	47.222
1 3 1 3	0.112	-0.334	40.000	31.000	45.667	52.667	38.444	40.111	57.667	47.222
2 3 1 4	1.000	-1.000	16.000	23.000	24.667	30.333	29.555	34.667	38.111	41.333
2 3 1 5	0.308	-0.555	22.000	23.000	28.667	31.333	29.555	34.667	37.556	41.333
2 3 1 6	2.421	1.556	31.000	23.000	35.333	42.333	29.555	34.667	48.333	41.333
1 3 2 1	3.572	-1.890	37.000	38.667	37.000	47.333	38.444	46.444	46.444	47.222
1 3 2 2	0.607	-0.779	32.000	38.667	32.667	36.667	38.444	46.444	37.556	47.222
1 3 2 3	7.108	2.666	47.000	38.667	45.667	55.333	38.444	46.444	57.667	47.222
2 3 2 4	1.777	1.333	23.000	26.333	24.667	35.000	29.555	38.000	38.111	41.333
2 3 2 5	0.308	-0.555	27.000	26.333	28.667	36.333	29.555	38.000	37.556	41.333
2 3 2 6	0.605	-0.778	29.000	26.333	35.333	42.667	29.555	38.000	48.333	41.333
1 3 3 1	3.158	1.777	46.000	45.667	37.000	54.333	38.444	55.111	46.444	47.222

Calculation of SDe for the PERIOD*DIAL and NOISE*PERIOD*DIAL Effects (continued):

N#P#D#S#	Yj^2	Yi =	Mnpds	-Mnpd.	-Mnp.s	-Mn.ds	+Mnp..	+Mn.d.	+Mn..s	-Mn...
1 3 3 2	0.308	0.555	41.000	45.667	32.667	46.000	38.444	55.111	37.556	47.222
1 3 3 3	5.448	-2.334	50.000	45.667	45.667	65.000	38.444	55.111	57.667	47.222
2 3 3 4	0.112	-0.334	35.000	39.333	24.667	49.000	29.555	51.333	38.111	41.333
2 3 3 5	1.234	1.111	37.000	39.333	28.667	45.000	29.555	51.333	37.556	41.333
2 3 3 6	0.605	-0.778	46.000	39.333	35.333	60.000	29.555	51.333	48.333	41.333
	127.111	-0.006								

$$\underline{SDe} = \text{Sqrt}\left(\sum_{j=1}^{18} (Yj\hat{}2) / \text{denominator } \underline{df} \right) = \text{SQRT}(127.111 / 16) = 2.818$$

Power Calculation for the PERIOD*DIAL effect:

COHEN	BAVRY
Effect Size = f = SDm / SDe	λ = SSm / MSe
Effect Size = f = 0.444 / 2.818	λ = 10.667 / (127.111/16)
Effect Size = 0.157	λ = 1.343
u = 4.0	DFm = 4.0
n' = (49 / (4 + 1)) + 1 = 10.8	DFe = 16.0
Effect power = 0.117	Effect power = 0.1068

Power Calculation for the NOISE*PERIOD*DIAL Effect:

COHEN	BAVRY
Effect Size = f = SDm / SDe	λ = SSm / MSe
Effect Size = f = 0.458 / 2.818	λ = 11.333 / (127.111/16)
Effect Size = 0.163	λ = 1.427
u = 4.0	DFm = 2.0
n' = (49 / (4 + 1)) + 1 = 10.8	DFe = 16.0
Effect power = 0.123	Effect power = 0.110

Note: Denominator df = (54/5 - 1) * 5 = 49

Appendix E

Calculation of the Average Correlation of Data from Appendix A

Correlation Matrices:

	P1 D1	P1 D2	P1 D3	P2 D1	P2 D2	P2 D3	P3 D1	P3 D2	P3 D3
P1 D1		.9315	.9567	.6881	.4275	.8151	.5789	.3888	.5396
P1 D2			.9514	.8737	.6726	.9195	.8066	.6321	.8021
P1 D3				.7116	.4163	.7959	.6693	.3922	.6568
P2 D1					.8655	.9157	.9665	.9183	.9212
P2 D2						.8019	.7890	.9188	.8402
P2 D3							.8332	.8072	.8464
P3 D1								.8928	.9554
P3 D2									.8642
P3 D3									

Fisher's Z Transform of Correlation Matrices*:

	P1 D1	P1 D2	P1 D3	P2 D1	P2 D2	P2 D3	P3 D1	P3 D2	P3 D3
P1 D1		1.669	1.905	0.844	0.457	1.142	0.661	0.410	0.604
P1 D2			1.847	1.348	0.815	1.586	1.117	0.745	1.104
P1 D3				0.890	0.443	1.087	0.810	0.414	0.787
P2 D1					1.315	1.561	2.036	1.578	1.597
P2 D2						1.104	1.069	1.581	1.222
P2 D3							1.198	1.119	1.243
P3 D1								1.435	1.890
P3 D2									1.310
P3 D3									

* Values were rounded to three significant digits.

Average Fisher's $\underline{Z}$ = 1.1652

$x = \exp_e (2 * 1.1652) = 10.28$

Average Correlation = (10.28 - 1) / (10.28 + 1) = .8226

Appendix F

Useful Conversion Algorithms

ϕ = SQRT(λ / (number of effect levels))

(Winer et al., 1991, p. 408)

ϕ = effect size * SQRT(the total number of observations on which the effect estimate is based)

(Winer et al., 1991, p. 409)

λ = effect size^2 * (the total number of observations on which the effect estimate is based)

(adapted from Cohen, 1988, p. 550)

λ = F * df1 Where: df1 = the numerator degrees of freedom

(J. L. Bavry, personal communication, September 14, 1995)

λ = (Sum of Squares Between) / (Mean Square Error)

(J. L. Bavry, personal communication, September 14, 1995)